Over & Back

Part 1:
Speed as a Rate

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Activity 1

1. You will set Rabbit's speed to 32 feet per second. Do not run Rabbit.

   What does it mean that Rabbit's speed is 32 feet per second?

- Predict how far Rabbit will go in 2 seconds.

  Prediction: *64 feet*

  Explanation: *If Rabbit runs at 32 ft/sec, then it will travel 32 feet in each of 2 seconds.*

- Predict how far Rabbit will go in 3 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 4 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 5 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 6 seconds.

  Prediction: _____

  Explanation:

Set Rabbit's speed to 32 ft/sec. Run Rabbit. Press **Pause** at each of these times (you probably will not stop it at precisely the desired time). Record the displayed time and displayed distance for each time you press **Pause**.

Pause 1 Displayed Time: _____ Displayed Distance: _____

Pause 2 Displayed Time: _____ Displayed Distance: _____

Pause 3 Displayed Time: _____ Displayed Distance: _____

Pause 4 Displayed Time: _____ Displayed Distance: _____

Pause 5 Displayed Time: _____ Displayed Distance: _____

Pause 6 Displayed Time: _____ Displayed Distance: _____

While answering the following questions, keep in mind that the computer's display does not show the *actual* time at which you stopped the timer or the *actual* distance Rabbit ran. Rather, it rounds actual time and actual distance to the nearest tenth, hundredth, or thousandth (depending on how many decimal places you've selected) before displaying them.

1) *For displayed times which match your target times*: Do the displayed distances match your predictions? If not, are they consistent with your predictions? Explain in detail. (Please respond on a separate piece of paper.)

2) *For displayed times which do not match your target times*: Are the displayed distances consistent with what "32 ft/sec" means? Explain in detail. (Please respond on a separate piece of paper.)

3) Julio ran Rabbit at 31.3 feet/sec and "Paused" it so that the timer displayed "3.0 seconds." The displayed distance was 92.4 ft. He figured that Rabbit did not go at the same speed all the time like his teacher said, since if it did it would have gone 93.9 ft.

Explain Julio's confusion.

4) Roberta ran Rabbit at 73 feet/sec and "Paused" it so that the displayed distance was 112.7 ft.

Within what bounds might Roberta *actually* have stopped the timer? (Be as precise as possible.)

Please comment on what you have learned:

2. You will set Rabbit's speed to 27 feet per second. Do not run Rabbit.

What does it mean that Rabbit's speed is 27 feet per second?

- Predict how far Rabbit will go in 2.3 second.

  Prediction: *62.1 feet*

  Explanation: *If Rabbit runs at 27 ft/sec, then it will travel 27 feet in each of 2 seconds, going 54 feet, and it will travel 3/10 of 27 feet in 3/10 of one second, going an additional 8.1 feet.*

- Predict how far Rabbit will go in 3.2 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 1.5 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 2.7 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 3.4 seconds.

  Prediction: _____

  Explanation:

- Predict how far Rabbit will go in 5.3 seconds.

  Prediction: _____

Explanation:

Set Rabbit's speed to 27 ft/sec. Run Rabbit. Press **Pause** at each of these times (you probably will not stop it at precisely the desired time). Record the displayed time and displayed distance for each time you press **Pause**.

Pause 1 Displayed Time: _____ Displayed Distance: _____

Pause 2 Displayed Time: _____ Displayed Distance: _____

Pause 3 Displayed Time: _____ Displayed Distance: _____

Pause 4 Displayed Time: _____ Displayed Distance: _____

Pause 5 Displayed Time: _____ Displayed Distance: _____

Pause 6 Displayed Time: _____ Displayed Distance: _____

While answering the following questions, keep in mind that the computer's display does not show the *actual* time at which you stopped the timer or the *actual* distance Rabbit ran. Rather, it rounds actual time and actual distance to the nearest tenth, hundredth, or thousandth (depending on how many decimal places you've selected) before displaying them.

1) For displayed times which match your target times: Do the displayed distances match your predictions? If not, are they consistent with your predictions? Explain in detail. (Please respond on a separate piece of paper.)

2) For displayed times which do not match your target times: Are the displayed distances consistent with what "27 ft/sec" mean? Explain in detail. (Please respond on a separate piece of paper.)

3) Use the data you collected while testing your predictions to complete this table. Construct the table, do your work, and explain your solutions on separate pieces of paper. *In explaining your solutions, be sure to say what quantity you intend to evaluate when performing a calculation.*

Speed: 27 ft/sec

Pause #	Displayed Distance	Range of Possible Actual Distances	Range of Possible Actual Times
1.			
2.			
3.			
4.			
5.			

6.

7. *x feet*

Please comment on what you have learned:

Activity 2

Turtle or Rabbit always take some amount of time when they run over and back. In this activity you will determine how much time they will take to go over and back at various speeds on a track that is 100 feet each way.

Here is an example of one student's reasoning. Try to use reasoning like this student's—it turns out to be very powerful for more complex situations.

> *Turtle is going to go over at 20 feet per second and back at 30 feet per second. On the way over, every time Turtle goes 20 feet it takes him 1 second. To find how many seconds it takes to go 100 feet, I need to find how many times he needs to travel 20 feet in order to go 100 feet (5 times, so 5 seconds to go over).*
>
> *On the way back, every time Turtle travels 30 feet it takes him 1 second. To find how many seconds it takes to go 100 feet I need to find how many times he needs to travel 30 feet in order to go 100 feet. He will go 30 feet 3 times (so 3 seconds to go 90 feet). The remaining 10 feet is 1/3 of 30 feet, so it will take him 1/3 of 1 second to travel it.*
>
> *So he will come back in 3 and 1/3 seconds. His total time is 8 and 1/3 seconds.*

	Turtle's Over Speed	**Turtle's Back Speed**	**Rabbit's Speed**	**Do This**
1.	20 ft/sec	30 ft/sec	--	Determine Turtle's time over and back.
2.	--	--	25 ft/sec	Determine Rabbit's time over and back.
3.	22 ft/sec	80 ft/sec	--	Determine Turtle's time over and back. Explain your reasoning.
4.	90 ft/sec	40 ft/sec	--	Determine Turtle's time over and back. Explain your reasoning.
5.	--	--	145 ft/sec	Determine Rabbit's time over and back. Explain your reasoning.
6.	123 ft/sec	34 ft/sec	--	Determine Turtle's time over and back. Explain your reasoning.
7.	42 ft/sec	18 ft/sec	25 ft/sec	Determine who will win. Explain your reasoning. (Use "0.000" when testing your prediction.)
8.	22 ft/sec	38 ft/sec	28 ft/sec	Determine who will win. Explain your reasoning. (Use "0.000" when testing your prediction.)

Activity 3

In this activity you will determine the speed at which Rabbit needs to move in order to complete a round trip in a specified time.

Part of this activity is that you explicitly realize relationships among time, distance, and speed. The reasoning expected of you in this activity builds on the reasoning emphasized in Activity 2, namely:

> At a speed of *x* ft/sec,
>
> (a) every time he goes *x* feet, he travels for 1 second,
>
> (b) every time he travels for one second he goes *x* feet,
>
> (c) if he goes some fraction of *x* feet he travels for that same fraction of 1 second,
>
> (d) every time he travels some fraction of 1 second, he goes that same fraction of *x* feet.

A generalization of these principles is:

> *If Rabbit goes some distance in some amount of time, then he will go a fraction of that distance in the same fraction of that amount of time. Conversely, if Rabbit travels for some amount of time and goes some distance, then if he goes for some fraction of that amount of time he will go the same fraction of that distance.*

1) Rabbit went 200 feet in 9 seconds. Restate the above generalization with this distance and time using 2/3, 3/8, and 1/9 in place of "some fraction." *[That is, restate it three times, once using 2/3, once using 3/8, and once using 1/9.]*

2) Confirm that your restated generalizations are accurate.

3) Use the italicized generalization to complete the following table. Complete it on a separate piece of paper.

Rabbit Will Run	In	Reasoning	Rabbit's Speed
200 ft	5.1 sec	Rabbit is supposed to run 200 feet in 5.1 seconds. To determine the speed at which he needs to run, I need to determine how far he travels in 1 second when he travels 200 feet in 5.1 seconds. One second is 1/5.1 of 5.1 seconds, so he must travel 1/5.1 of 200 feet in 1 second to go 200 feet in 5.1 seconds.	39.22 ft/sec
200 ft	11 sec		
450.38 ft	8 sec		
270.8 ft	7 sec		
156 ft	6.5 sec		
422.57 ft	7.2 sec		
334.86 ft	8.3 sec		

Activity 4

Figure out what speeds to put in the blanks so that Turtle and Rabbit will tie. Use a separate piece of paper on which to perform your calculations and to explain your reasoning.

Turtle->	**<-Turtle**	**Rabbit**	**Distance One Way**
20	40		100
	70	30	100
52		40	100
21	48		200
	70	30	200
52		40	225
31		55	682

After you have figured what numbers to put in the blanks, do this:

1) Describe the arithmetic you will do tomorrow when you are given Turtle's over-speed, Turtle's back-speed, and the length of the track, and you are asked to enter a number for Rabbit's speed that will make Turtle and rabbit tie. *Please state initial conditions, and say what you are finding whenever you say to do a calculation.*

2) Describe the arithmetic you will do tomorrow when you are given Turtle's over-speed, Rabbit's speed, the length of the track, and you are asked to find a number for Turtle's back-speed that will make Rabbit and turtle tie. *Please state initial conditions, and say what you are finding whenever you say to do a calculation.*

Activity 5

This activity is in two parts—-one that you do at your desk and one that you do at the computer. The part that you do at your desk will be to make predictions about what will happen when you go to the computer. The part that you do at the computer will be to collect data on your predictions.

Up to this time you have been predicting how far Rabbit will go in some number of seconds or how many seconds it will take Turtle to go some distance. This activity asks you to think about the how much farther one animal has run than the other and to think about how fast that distance changes.

Part I (At your desk.)

Do whatever arithmetic you need to do in order to calculate the requested values for time or for the distance between the animals. Attach your (neat, organized, and clear) work to this page.

Predictions

Distance (one-way)	*Turtle's Over Speed*	*Turtle's Back Speed*	*Rabbit's Speed*	*Number of Seconds*	*Difference in Total Distances*	*Rate at which Diff in Total Is Changing*
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	1 sec		
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	2 sec		
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	3 sec		
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	4 sec		
150 ft	46 ft/sec	40 ft/sec	27 ft/sec	4.7 sec		
210 ft	81 ft/sec	32 ft/sec	65 ft/sec	3.8 sec		
200 ft	26 ft/sec	31 ft/sec	80 ft/sec		250 ft	
200 ft	72 ft/sec	10 ft/sec	48 ft/sec		27 ft	
200 ft	11 ft/sec	500 ft/sec	20 ft/sec		10 ft	

Part II (Test predictions)

Now, run both Rabbit and Turtle. Stop them at the appropriate moments so that you can test your predictions. Write the actual time and the actual distance between them at the moments you press **Pause**.

Observations

Distance (one-way)	*Turtle's Over Speed*	*Turtle's Back Speed*	*Rabbit's Speed*	*Number of Seconds*	*Difference in Total Distances*	*Rate at which Diff in Total Is Changing*
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	1 sec		
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	2 sec		
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	3 sec		
100 ft	32 ft/sec	40 ft/sec	36 ft/sec	4 sec		
150 ft	46 ft/sec	40 ft/sec	27 ft/sec	4.7 sec		
210 ft	81 ft/sec	32 ft/sec	65 ft/sec	3.8 sec		
200 ft	26 ft/sec	31 ft/sec	80 ft/sec		250 ft	
200 ft	72 ft/sec	10 ft/sec	48 ft/sec		27 ft	
200 ft	11 ft/sec	500 ft/sec	20 ft/sec		10 ft	

Part III

You may have noticed that, at any moment during a race, the rate at which the difference in the animals' total distances is changing appears to be identical to the difference of their speeds at that moment.

I. Is that always the case? If so, why? If not, why not?

Over & Back

Part 2:
Rates in General

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Here is a situation:

John traveled 35 miles per hour for 138 miles and 44 miles per hour for 50 miles.

• How is this situation like turtle, rabbit, or turtle and rabbit situations?

John is like:

35 miles per hour is like:

138 miles is like:

44 miles per hour is like:

50 miles is like:

• How is this situation unlike turtle, rabbit, or turtle and rabbit situations?

The following question can be answered with only one calculation if only you could figure out something about this situation. What would you need to figure out in order to answer this question with just one calculation? *[Don't actually answer the question.]*

Question:	How fast would John need to travel if he were to go at a constant speed and go just as far in the same amount of time as before?

What you would need to figure out, and what calculation you would perform after you have it:

Here is a situation:

Sue paid $9.46 for Yummy candy bars, and she paid $6.08 for Zingy candy bars. One Yummy candy bar costs $0.43. Sue bought 38 candy bars altogether.

• How is this situation like turtle, rabbit, or turtle and rabbit situations?

Sue is like:

$9.46 is like:

$6.08 is like:

No. of Zingy bars is like:

No. of Yummy bars is like:

$0.43 is like:

38 bars is like:

• How is this situation different from turtle, rabbit, or turtle and rabbit situations?

The following question can be answered with only one calculation if only you could figure out something about this situation. What would you need to figure out in order to answer this question with just one calculation? *[Don't actually answer the question.]*

Question: What was the price of a Zingy candy bar?

What you would need to figure out, and what calculation you would perform after you have it:

Here is a situation:

Janna has a swimming pool that holds 93,000 gallons of water. She began using one pump to fill her pool, but it broke down when the pool was one-third full and she had to replace it. Her first pump put water into the pool at 73 gallons per minute. The second pump put water into her pool at 68 gallons per minute.

• How is this situation like turtle, rabbit, or turtle and rabbit situations?

93,000 gallons of water is like:

First pump is like:

One-third full is like:

Second pump is like:

73 gallons per minute is like:

68 gallons per minute is like:

• How is this situation different from turtle, rabbit, or turtle and rabbit situations?

The following question can be answered with only one calculation if only you could figure out something about this situation. What would you need to figure out in order to answer this question with just one calculation? *[Don't actually answer the question.]*

Question: How long did it take Janna to fill her swimming pool?

What you would need to figure out, and what calculation you would perform after you have it:

Here is a situation:

Janna has a swimming pool that holds 93,000 gallons of water. She began using one pump to fill her pool, but it broke down and she had to replace it. Her first pump put water into the pool at 73 gallons per minute. The second pump put water into her pool at 68 gallons per minute. It took 1321 minutes of pumping time to fill her pool.

• How is this situation like turtle, rabbit, or turtle and rabbit situations?

93,000 gallons is like:

First pump is like:

73 gallons per minute is like:

Second pump is like:

68 gallons per minute is like:

1321 minutes is like:

• How is this situation different from turtle, rabbit, or turtle and rabbit situations?

• How is this different from the first "Janna's Pool" situation?

The following question can be answered with only one calculation if only you could figure out something about this situation. What would you need to figure out in order to answer this question with just one calculation? *[Don't actually answer the question.]*

Question: How long did each pump work?

What you would need to figure out, and what calculation you would perform after you have it:

Here is a situation:

Carol has a swimming pool that holds 42,500 gallons of water. She filled it with two pumps running at the same time. One pump put water into the pool at 45 gallons per minute. The other pump put water into the pool at 70 gallons per minute.

• How is this situation like turtle, rabbit, or turtle and rabbit situations?

42,500 gallons of water is like:

First pump is like:

Second pump is like:

45 gallons per minute is like:

70 gallons per minute is like:

• How is this situation different from turtle, rabbit, or turtle and rabbit situations?

The following question can be answered with only one calculation if only you could figure out something about this situation. What would you need to figure out in order to answer this question with just one calculation? *[Don't actually answer the question.]*

Question:	How long will it take Carol to fill her swimming pool using both pumps at the same time if the pool is now empty?

What you would need to figure out, and what calculation you would perform after you have it:

Here is a situation:

Jim has a swimming pool that holds 35,000 gallons of water. He has two pumps running at the same time. One pump can fill his pool in 912 minutes. The other pump can fill his pool in 532 minutes.

• How is this situation like turtle, rabbit, or turtle and rabbit situations?

35,000 gallons of water is like:

First pump is like:

Second pump is like:

912 minutes is like:

532 minutes is like:

• How is this situation different from turtle, rabbit, or turtle and rabbit situations?

• How is it different from the "Carol's" pool situation?

The following question can be answered with only one calculation if only you could figure out something about this situation. What would you need to figure out in order to answer this question with just one calculation? *[Don't actually answer the question.]*

Question:	How long will it take Jim to fill his swimming pool using both pumps at the same time if the pool is now empty?

What you would need to figure out, and what calculation you would perform after you have it:

Over & Back Follow-up

1. A car traveled the distance from City *A* to City *B* at 20 km per hour, and traveled back at 30 km per hour. How fast would this car need to go to make the same trip in the same amount of time while going at a constant speed?

   a. How is this similar to situations you've encountered previously? How is it different?
   b. Answer the question.

   The distance between the cities is not given. Pick several distances and solve the problem once for each distance. (i) What do you notice? (ii) Explain why this happens.

2. This is an ancient problem: A man drank a tub of water over 14 days, and he and his wife drank one over 10 days. How much time would the wife alone take to drink one tub of water?

   a. How is this similar to situations you've encountered previously? How is it different?
   b. Answer the question.

3. A steamer travels at a constant speed in still water. Tomorrow it will go from Astakhan to Volgograd and back. Will this journey be accomplished faster if there is a current or if there is not a current?

   a. How is this similar to situations you've encountered previously? How is it different?
   b. Answer the question.

4. One typist can type 1,000 bibliography cards in 9 hours. A second typist can type 1,000 cards in 6 hours. How many hours will they take to get 1,000 typed cards if they work on this job together?

   a. How is this similar to situations you've encountered previously? How is it different?
   b. Answer the question.