Three-dimensional graphs

The Graphing Calculator can graph several forms of three-dimensional equations.

Graph	Example
Explicit function surface plot	$z = \cos xy$
Implicit surface	$x^2 + y^2 + z^2 = 1$
Parametric curve	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos 4\pi t \\ \sin 4\pi t \\ 2t \end{bmatrix}$
Parametric surface	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2u - 1 \\ 2v - 1 \\ \cos 4uv \end{bmatrix}$
Vector field	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z \\ x \\ y \end{bmatrix}$
Point	$\begin{bmatrix} 1 \\ 2 \\ n \end{bmatrix}$
3D Coordinate transformation	$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = 2\begin{bmatrix} x \\ y \\ z \end{bmatrix}$
Graphs involving complex numbers	
Complex parametric curve	$x + iy = e^{2\pi iz}$
Complex surface	$\mathrm{Re}\,(\sin z)$

Explicit function surface plots

The calculator will draw surfaces from equations in any of the following forms:

$z = f(x,y)$ $z = f(r,\theta)$ $r = f(\theta,\Phi)$

$y = f(x,z)$ $r = f(\theta,z)$ $\theta = f(r,\Phi)$

$x = f(y,z)$ $\theta = f(r,z)$ $\Phi = f(r,\theta)$

The surface is drawn for the ranges of the independent coordinates shown on the right side of the math pane. You can , if desired.

Examples:

$z = \cos xy$

$y = \exp{-(x^2 + z^2)}$

$x = \text{sech}\, 2y + \text{sech}\, 2z$

$z = \cos r + \sin \theta$

$r = \cos (z + \theta)$

$\theta = zr$

$r = 2 \sin^2 \Phi \cos 2\theta$

$\theta = 4 \cos^2 \Phi + r$

$\Phi = 1 + \sin r + \sin 3\theta$

The calculator will also recognize functions of the form $z = f(x,y,r,\theta)$, where r is a cylindrical coordinate. In this case, r and θ are not independent coordinates.

Example: $z = \sin r \sin 3\theta + \cos xy$

For surfaces of the form $z = f(x,y)$, except for the cases $z = f(x)$ and $z = f(y)$, you can omit "z =" and type just the expression on the right hand side. You can also omit "z =" for surfaces of the form $z = f(r,\theta)$ and of the form $z = f(x,y,r,\theta)$.

Implicit surfaces

The calculator will draw a surface defined by an implicit relation in coordinates x, y and z.

Examples:

$x^2 + y^2 + z^2 + \sin 2x + \sin 2y + \sin 2z = 2$

$z - 2 = \sin xy$

Even though this equation can be written in the explicit form $z = \sin xy + 2$, the calculator will consider it an implicit relation.

Implicit surfaces are drawn only in the *x*, *y*, and *z* ranges shown at the right side of the math pane.

Neither the extended color menu nor texture maps are available for implicit surfaces.

Parametric curves

A parametric curve is a locus of points whose coordinates are given as functions of a parameter *t*. You can write equations in the following forms:

General Form	Example

Draw the curve traced by the equations $x = f(t)$, $y = g(t)$, and $z = h(t)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix} \qquad\qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ t \end{bmatrix}$$

Draw the curve traced by the equations $r = f(t)$, $\theta = g(t)$, and $z = h(t)$

$$\begin{bmatrix} r \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix} \qquad\qquad \begin{bmatrix} r \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 2 \cos 2\pi t \\ \sin 2\pi t \\ 2t(2 - t^2) \end{bmatrix}$$

Draw the curve traced by the equations $r = f(t)$, $\theta = g(t)$, and $\Phi = h(t)$

$$\begin{bmatrix} r \\ \theta \\ \Phi \end{bmatrix} = \begin{bmatrix} f(t) \\ g(t) \\ h(t) \end{bmatrix} \qquad\qquad \begin{bmatrix} r \\ \theta \\ \Phi \end{bmatrix} = \begin{bmatrix} 3t \\ 5t \\ 7t \end{bmatrix}$$

The range of *t* for which the curve is drawn is shown at the right side of the math pane. You can change this range, if desired, by editing it manually. The default range is 0 to 1.

Parametric surfaces

The set of points whose coordinates are given as functions of two parameters *u* and *v* make a parametric surface. You can write equations in the following forms:

General Form	Example

Draw the surface defined by the equations $x = f(u,v)$, $y = g(u,v)$, and $z = h(u,v)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f(u,v) \\ g(u,v) \\ h(u,v) \end{bmatrix} \qquad\qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (1 + v \sin 2\pi u) \sin 2\pi v \\ v \cos 2\pi u \\ (1 + v \sin 2\pi u) \cos 2\pi v \end{bmatrix}$$

Draw the surface defined by the equations $r = f(u,v)$, $\theta = g(u,v)$, and $z = h(u,v)$

$$\begin{bmatrix} r \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} f(u,v) \\ g(u,v) \\ h(u,v) \end{bmatrix} \qquad \begin{bmatrix} r \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} 2 + \cos 2\pi v \\ 2\pi u \\ \sin 2\pi v \end{bmatrix}$$

Draw the surface defined by the equations $r = f(u,v)$, $\theta = g(u,v)$, and $\Phi = h(u,v)$

$$\begin{bmatrix} r \\ \theta \\ \Phi \end{bmatrix} = \begin{bmatrix} f(u,v) \\ g(u,v) \\ h(u,v) \end{bmatrix} \qquad \begin{bmatrix} r \\ \theta \\ \Phi \end{bmatrix} = \begin{bmatrix} 1 + \cos \pi v + \sin \pi u \\ 2\pi u(v^v + u^v) \\ \pi u \end{bmatrix}$$

The ranges of u and v for which the surface is drawn appear on the right side of the math pane. You can can change these ranges. The default ranges of u and v are both 0 to 1.

Vector fields

To draw a three-dimensional vector field, define the x, y, and z components as functions of the coordinates x, y, and z by writing an equation in the form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} f(x,y,z) \\ g(x,y,z) \\ h(x,y,z) \end{bmatrix}$$

Example: Draw the vector field

$\mathbf{V}(x,y,z) = f(x,y,z)\,\mathbf{i} + g(x,y,z)\,\mathbf{j} + h(x,y,z)\,\mathbf{k}$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors in the x, y, and z directions respectively

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -z \\ y \end{bmatrix}$$

For every point in space, this defines a vector with x component one, y component equal to minus the z coordinate of the point, and z component equal to the y coordinate of the point.

The calculator evaluates the vector field on a regular lattice of points filling the graph region. Vectors are drawn with their bases at these points, and their lengths are scaled so that the longest vector fits between adjacent lattice points. Vectors with infinite or undefined components are not drawn.

It can be visually confusing when 3D vectors are drawn at every point in a cubic lattice.

You can limit drawing to only vectors on a surface. Any equation for an explicit function surface plot, implicit surface, or parametric surface, can be written after the equation for a vector field, separated by a comma. The surface itself won't be displayed, but the vector field will only be drawn at points on the surface.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.1 \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ r = 3$$

Shows an outward pointing vector field on the surface of a sphere.

Points

The calculator can graph a single point in space by specifying its x, y, and z coordinates. You specify coordinates by writing an equation in the form:

$$\begin{bmatrix} \# \\ \# \\ \# \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

3D coordinate transformations

The calculator can visualize 2D and 3D coordinate transformations. It draws side-by-side graphs with the untransformed, unprimed coordinates on the left and the transformed, primed coordinates on the right.

Example: Draw a sphere of radius one on the left and scale it by a factor of two on the right.

Enter and graph the following two equations together:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = 2 \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x^2 + y^2 + z^2 = 1$$

Complex parametric curves

The calculator can plot complex functions of a real parameter as a curve in three dimensions.

You can specify the curve by writing an equation in the form:

$x + iy = f(z)$

which is almost equivalent to:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \text{Re } f(t) \\ \text{Im } f(t) \\ t \end{bmatrix}$$

except that the range of z is determined by the view.

Example:

$x + iy = e^{2\pi iz}$

Complex surfaces

The calculator can plot complex functions as surfaces in three dimensions by specifying a real part using Re, Im, arg, or ||.

You can specify the curve by writing an equation in the form:

$f(z)$

where $f(z)$ evaluates to a real number. This is equivalent to:

$z = f(x + iy)$

Examples:

Re $(\sin z)$

Im $(\sin z)$

$|\sin z|$

Re $(\sin z \cdot \sin \pi n)$ + Im $(\sin z \cdot \sin \pi n)$